



Development of a Simpler but Accurate Free Gas Deviation Factor for Fractured (Dual-Porosity) Volumetric Gas Reservoir Fluid

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Authors' contributions

This work was carried out in collaboration between both authors. Authors AJA and OAF designed the study. Author AJA performed the statistical analysis and wrote the protocol and the first draft of the manuscript under the supervision of author OAF. Both authors managed the literature searches, the analyses of the study, read and approved the final manuscript.

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ABSTRACT

Gas compressibility factor, also known as gas deviation factor or Z-factor, is a thermodynamic correction factor which describes the deviation of a real gas from ideal gas behaviour. The free gas Z-factor in the Material Balance Equation (MBE) of single-porosity gas reservoirs with insignificant rock (matrix) compaction (after pressure depletion) does not reflect cases in low-permeability gas reservoirs having remarkable rock compaction. Through gas MBE modifications, previous researchers developed Z-factors for dual-porosity (fractured) low permeability gas reservoirs by incorporating gas desorption; however, their approaches create complexity for routine calculations. Therefore this study was designed with the purpose of deriving a free gas Z-factor for single-porosity low-permeability gas reservoirs and further modifying it for more simplicity and accuracy in a dual-porosity scenario. The free gas Z-factor derived for single-porosity low-permeability gas reservoirs is expressed as: $Z^* = Z \cdot \left\{ 1 - \left(\frac{C_w S_{wi} + C_{matrix}}{S_{g_i}} \right) \Delta P \right\}^{-1}$ where Z , C_w , S_{wi} , C_{matrix} , S_{g_i} and ΔP are single-porosity Z-factor without rock compaction at pressure P , water

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compressibility, initial water saturation, matrix compressibility, initial gas saturation and pressure depletion, respectively. However, the developed dual porosity free gas Z-factor model incorporates ratio of dual porosity to initial matrix porosity, and it is expressed as: $Z^{**} = Z \cdot \left\{ 1 - \left(1 - \phi_{frac} + \frac{\phi_{frac}}{\phi_{mat}} \right) \left(\frac{c_w S_{wi} + c_{matrix}}{S_{gi}} \right) \cdot \Delta P \right\}^{-1}$ where ϕ'_{mat} and ϕ_{frac} are initial matrix porosity and fracture porosity, respectively. The Z-factor model was graphically and statistically correlated with an existing free gas Z-factor model for dual porosity reservoirs. For all the hydraulically fractured shale gas formations considered, the correlations yield R^2 values of 1.000.

Keywords: Single-porosity; dual-porosity; free gas; Z-factor; hydraulically fractured gas reservoirs.

1. INTRODUCTION

The concept of an ideal gas is based on the assumptions that the molecule's kinetic energy is more remarkable than the potential energy due to intermolecular attractive and repulsive forces (because the collision between the particles are assumed to be elastic), and that the size of the molecules is insignificant compared to the empty space between them. This is usually attained at higher temperature and lower pressure. However, a gas behaves more like a real gas at lower temperature and higher pressure where the gas is getting close to a phase change, and the kinetic theory and the relative size assumptions do not hold.

Gas compressibility factor, also known as gas deviation factor or Z-factor, is a thermodynamic correction factor which describes the deviation of a real gas from ideal gas behaviour. Z-factor values are usually obtained through experiments or by calculation from equation of state (EoS) featuring pressure P , volume V , temperature T and number of moles n [1-3]. Gas compressibility c should not be confused with the compressibility factor (Z-factor). The compressibility of a substance is defined as the change in volume per unit volume per unit change in pressure.

Z-factor determination through laboratory PVT analyses are sometimes expensive and time consuming. Hence, charts, correlations and other mathematical methods are often used to predict Z-factor because they are much easier and faster than equations of state thus save cost. Standing and Katz [4] presented a generalized natural gas Z-factor chart as a function of the pseudo-reduced pressure and pseudo-reduced temperature. The ranges considered are $1.05 \leq T_{pr} \leq 3.0$ and $0 \leq P_{pr} \leq 15$.

For the purpose of improving accuracy, many empirical equations have been fit to the original

Standing and Katz chart. These correlations are: Wichert and Aziz correlation [5] (that considers effect of H_2S and CO_2 contaminants), Hall and Yarborough correlation [6] (that offers an accurate representation of the Standing and Katz chart), Beggs and Brill's correlation [7], Dranchuk-Purvis-Robinson correlation [8], Dranchuk and Abou-Kassem [9] (that fits an eleven-constant EoS to the Standing and Katz data, and extrapolated this correlation to higher reduced pressures of the range $0 \leq P_{pr} \leq 20$) and Takacs correlation [10].

Based on Takacs [10] comparison of eight correlations representing the Standing and Katz chart [5], the Hall and Yarborough [6], and the Dranchuk and Abou-Kassem [9] equations give the most accurate representation for a wide range of temperatures and pressures. Both equations are valid for $1 \leq T_{pr} \leq 3$ and $0.2 \leq P_{pr} \leq 25$ to 30. The Hall and Yarborough or Dranchuk and Abou-Kassem equation is recommended for the evaluation of the Z-factor of most natural gases. For sour gas, gas deviation factor can be calculated using Piper *et al.* correlation [11], and Elsharkawy and Elkamel correlation [12].

The correlations developed for calculating pseudo-critical properties (pseudo-critical pressure and pseudo-critical temperature) are Standing correlation [13] developed only for low molecular weight natural gases with minor amount of non-hydrocarbon gases; Sutton correlation [14] for a wide range of natural gas (it considers high molecular weight natural gases which are rich in heptane plus with minor concentration of carbon dioxide and nitrogen, and no hydrogen sulphide); Wichert-Aziz and Carr-Kobayashi-Burrows correction methods [15] applicable for mixture with impurities such as N_2 , CO_2 , H_2S ; Elsharkawy *et al.* correlation [16] suitable for retrograde gases (gas condensates); and Guo and Ghalambor [17] correlation valid for

H₂S < 3%, N₂ < 5%, and total content of inorganic compounds less than 7%.

Standing [13] correlation gives consideration to both dry and wet hydrocarbon gases. For dry hydrocarbon gases ($\gamma_g < 0.75$) the correlation is stated as:

$$P_{pc} = 667 + 15.0\gamma_g - 37.5\gamma_g^2 \quad (1)$$

$$T_{pc} = 168 + 325\gamma_g - 12.5\gamma_g^2 \quad (2)$$

For wet hydrocarbon gases ($\gamma_g \geq 0.75$) the correlation is stated as:

$$P_{pc} = 706 + 51.7\gamma_g - 11.1\gamma_g^2 \quad (3)$$

$$T_{pc} = 187 + 330\gamma_g - 71.5\gamma_g^2 \quad (4)$$

where P_{pc} is in psi, T_{pc} is in °R and γ_g is the gas specific gravity which is expressed as the ratio of gas molecular weight to air molecular weight.

Sutton correlation [14] is valid for $0.57 < \gamma_g < 1.68$ based on 264 different gas samples. Pseudo-critical pressure and pseudo-critical temperature in Sutton correlation are stated as:

$$P_{pc} = 756.8 - 131.0\gamma_g - 3.6\gamma_g^2 \quad (5)$$

$$T_{pc} = 169.2 + 349.5\gamma_g - 74.0 \quad (6)$$

Given the size of the database used in its development, Sutton's correlation is a good representative for all gas reservoirs [18].

Shale gas processing is often guided by gas compositions across a field [19]. Therefore, ethane and propane levels should be considered in choosing either Standing correlation [13] or Sutton correlation [14] for shale gas pseudo-critical properties evaluation.

Many research works have been done in determining Z-factor experimentally or evaluating it through different Z-factor correlations and other mathematical methods. Adeleye and Olamigoke [20] used Artificial Neural Networks (ANN) in predicting Z-factor. The ANN model correlates gas compressibility factor as a function of reservoir temperature and dew point pressure. ANN was applied to the 40 raw data sets in the range of 105-226 °F and 2445-4843 psia for temperature, and pressure, respectively.

Also, Obuba et al. [21] presented a new natural gas Z-factor correlation for Niger Delta gas fields

by developing gas properties databank from twenty-two laboratory gas PVT reports from the fields (comprising dry gas, solution gas, rich CO₂ gas and rich condensate gas reservoirs). The developed correlation yielded better match than the existing correlations considered.

Using samples of gas mixtures from high-pressure-high-temperature (HPHT) gas reservoirs in the Niger Delta region of Nigeria, Azubuike et al. [22] carried out laboratory measurement of Z-factors. Most of the Z-factor correlations studied were found to overestimate the gas compressibility factor at HPHT. However, Beggs and Brill's correlation [7] was reported to predict better than other correlations with a mean relative error of -4.77 and absolute error of 7.187.

Hamada [23] studied the effect of non-hydrocarbon components on Z-factor values using correlations. Gas pseudo-critical temperature was found to decrease with increase of N₂ and H₂S. Also, some deviations in Z-factor estimation were observed when using Stewart mixing rule and Kay's mixing rule for gas reservoirs containing C7+. However, these deviations were negligible when correlation considering non-hydrocarbon was used.

Also, Mamedov *et al.* [24] presented an analytical model for evaluating the Z-factor of real gases such as H₂, N₂, He, CO₂, CH₄ and air at various temperatures using the second virial coefficient with a Lennard-Jones (12-6) potential. Results indicated good match with the data in the literature.

Akinsete and Omotosho [25] modeled two-phase gas Z-factor for gas-condensate reservoir using Artificial Neural Network. The two-phase gas Z-factor obtained in ANN method gave the closest value to the observed two-phase gas Z-factor from laboratory work.

1.1 Basis for Z-Factor Modification for Dual-Porosity Gas Reservoirs

The Z-factors evaluated experimentally or through different Z-factor empirical correlations and other mathematical methods represent cases of gas flow in single-porosity natural gas reservoirs with insignificant pore compaction after pressure depletion. The Z-factor does not reflect the remarkable rock compaction phenomenon in low-permeability natural gas reservoirs with single-porosity and dual-porosity.

1.2 Z-Factor Modification for Dual-Porosity Gas Reservoirs using Gas Material Balance Equation (MBE)

Schilthuis [26] presented an equation called material balance equation (MBE), formulated to consider the reservoir as a single tank characterised by homogeneous rock properties. The equation keeps inventory of all materials entering, leaving and accumulating in the reservoir and was developed as a volume balance in which the underground withdrawal in the form of observed cumulative production is equal to the expansion of the fluids in the reservoir due to pressure depletion. MBE thereafter became a basic tool often used by reservoir engineers to interpret and predict reservoir performance. When appropriately used, MBE can be utilised to evaluate initial hydrocarbon volumes in place, forecast reservoir production performance, and predict oil and gas recovery under different types of primary driving mechanisms [27].

If there is negligible rock compaction in a single-porosity volumetric reservoir (no water influx or water production), the traditional gas MBE is expressed as:

$$\frac{P}{Z} = \frac{P_i}{Z_i} \left(1 - \frac{G_p}{G}\right) \quad (7)$$

i.e.

$$\frac{G_p}{G} = 1 - \frac{P/Z}{P_i/Z_i} \quad (8)$$

where G_p is cumulative gas produced (scf), G is free gas initially in place (scf), P is reservoir pressure, Z is gas deviation factor (Z-factor), and subscript i denoted condition before expansion.

Researchers that previously worked on modifying Z-factor in single-porosity gas MBE, for dual-porosity gas reservoirs, include King [28], Aguilera [29], Moghadam et al. [30], and Duarte et al. [31].

King [28] modified the MBE for original gas-in-place (OGIP) evaluation and future performance prediction for wells in coalbed and Devonian shale reservoirs. In the approach, equilibrium condition was assumed for free and adsorbed gases. Also, gas desorption from the matrix blocks to the fracture system was assumed to be in pseudo-steady state. King's method works just like the traditional MBE, where the straight line

plot of P/Z versus cumulative production G_p is used in estimating OGIP.

The MBE presented by King [28] is expressed as:

$$\left(\frac{G_p}{G}\right)_{Total\ Gas} = 1 - \frac{P/Z}{P_i/Z_i} \quad (9)$$

where $Z^* = Z / \left(S_g + \frac{V_L T P_{sc} Z}{\phi(P+P_L^* Z_{sc})} \right)$ with the assumption that rock and fluid compressibilities are negligible and water saturation is constant, P is average reservoir pressure, Z is Z-factor, Z_{sc} is Z-factor at surface condition, S_g is gas saturation, T is reservoir temperature, T_{sc} is temperature at surface condition, P is reservoir pressure, P_{sc} is pressure at surface condition, V_L is Langmuir volume (maximum adsorbed gas) and P_L^* is Langmuir pressure (at $V_L/2$). A plot of P/Z^* versus cumulative production G_p yields a straight line and can be extrapolated to evaluate OGIP.

For fractured gas reservoirs without gas adsorption, Aguilera [29] incorporated a dual porosity model where a tank is considered for the matrix pores and another for the fracture systems. Thus, gas material balance equation for fractured gas reservoirs where the effect of gas desorption on production was not considered yields:

$$\left(\frac{G_p}{G}\right)_{Free\ Gas} = 1 - \frac{P/Z'}{P_i/Z_i} \quad (10)$$

where $Z' = Z / \left\{ 1 - \left((1 - \omega_f) C' + \omega_f C'' \right) \Delta P \right\}$ is the Aguilera [29] dual-porosity free gas Z-factor, Z is single-porosity Z-factor at pressure P without pore compaction ω_f is OGIP (free gas) fraction within the fractures, $(1 - \omega_f)$ is OGIP (free gas) fraction within the matrix blocks, ΔP is change in pressure, $C' = \frac{C_{pm} + C_w S_{wm}}{(1 - S_{wm})}$, $C'' = \frac{C_f + C_w S_{wf}}{(1 - S_{wf})}$, C_w is water compressibility, S_{wm} is water saturation in the matrix and S_{wf} is water saturation in the fracture system. The details of the derivation of Aguilera [29] dual-porosity free gas Z-factor is shown in Appendix.

Moghadam et al. [30] improved on the MBE presented by King [28] and obtained a normalised compressibility factor expressed as:

$$Z^{**} = Z \frac{Z_i}{Z_i^*} \quad (11)$$

The plot of P/Z^{**} versus G_p has resemblance with the traditional P/Z versus G_p . The modified material balance equation was reported to be applicable to all kinds of gas reservoirs i.e., unconventional, over-pressure and water-driven. Cumulative gas production G_p can thus be evaluated as:

$$G_{p\ Total} = \frac{\phi V_B Z_i^2}{B_{g_i} P_i Z_i^*} \left\{ \frac{P_i}{Z^{**}} - \frac{P}{Z^{**}} \right\} \quad (12)$$

The estimated values of G_p were then correlated with G_p values obtained from shale gas production analysis (SGPA) by plotting P/Z versus G_p , and G_p versus time. The results showed good match.

However, Duarte et al. [31] incorporated gas desorption into the Aguilera [29] MBE for fractured gas reservoirs to yield:

$$\left(\frac{G_p}{G} \right)_{Total\ Gas} = 1 - \frac{P/Z_c}{P_i/Z_i} \quad (13)$$

where

$$Z_c = Z \left\{ 1 - \omega_a - (\omega_m C_{pm}^i + \omega_f C_f^i) \Delta P + \omega_m \frac{\rho_b B_g}{35.515 \phi (1 - S_{wm})} V_{ads} \right\}^{-1} \quad (14)$$

and $\omega_a = \frac{OGIP_a}{OGIP_{Total}}$ is the ratio of the adsorbed gas to the original gas-in-place, $\omega_m = \frac{OGIP_m}{OGIP_{Total}}$ is the fraction of the original gas-in-place that is initially stored within the matrix pores, $\omega_f = \frac{OGIP_f}{OGIP_{Total}}$ is the fraction of the original gas-in-place that is initially stored within the fracture network, ρ_b is shale bulk density (g/cm^3), ϕ is porosity and V_{ads} is the adsorbed gas volume. Duarte et al. [31] evaluated V_{ads} using the Langmuir adsorption isotherm $V_{ads} = V_L \frac{P}{P_L + P}$ where V_L is Langmuir volume (scf/ton), P is average reservoir pressure (psia) and P_L is Langmuir pressure (psia).

It is thus observed that King [28], Moghadam et al. [30] and Duarte et al. [31] developed Z-factors into which gas desorption was lumped, rendering them complex for routine calculations because cumulative free gas production should feature Z-factor while cumulative gas desorption should feature adsorption isotherm. However, Aguilera [29] developed a dual-porosity free gas Z-factor that incorporates OGIP fractions within fractures and matrix pores.

Total gas production is the sum of cumulative free gas production and cumulative gas

desorption, i.e. $G_{p\ Total} = G_{p\ free} + G_{p\ desorbed}$. Therefore, in this work, free gas Z-factor is derived for single-porosity reservoirs with remarkable pore compaction, and it is further modified to a simpler but accurate dual-porosity free gas Z-factor that is statistically correlated with Aguilera [29] free gas Z-factor.

2. METHODOLOGY

2.1 Evaluation of Z-Factor

Three shale gas formations A, B and C are considered. As stated earlier, the choice of using either Standing correlation [13] or Sutton correlation [14] for shale gas pseudo-critical properties evaluation depends on ethane and propane levels consideration.

The pseudo-reduced pressure P_{pr} and pseudo-reduced temperature T_{pr} are then calculated as $P_{pr} = P/P_{pc}$ and $T_{pr} = T/T_{pc}$. Thereafter, the single-porosity Z-factors at different pressure levels without pore compaction are evaluated using the Dranchuk-Abou-Kassem eleven-constant equation of state (EoS) [9] expressed as:

$$Z = 1 + C_1(T_{pr}) \cdot \rho_r + C_2(T_{pr}) \cdot \rho_r^2 - C_3(T_{pr}) \cdot \rho_r^5 - C_4(\rho_r, T_{pr}) \quad (15)$$

where

$$\rho_r = \frac{0.27 P_{pr}}{Z T_{pr}} \quad (16)$$

$$C_1(T_{pr}) = A_1 + \frac{A_2}{T_{pr}} + \frac{A_3}{T_{pr}^3} + \frac{A_4}{T_{pr}^4} + \frac{A_5}{T_{pr}^5} \quad (17)$$

$$C_2(T_{pr}) = A_6 + \frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2} \quad (18)$$

$$C_3(T_{pr}) = A_9 \left(\frac{A_7}{T_{pr}} + \frac{A_8}{T_{pr}^2} \right) \quad (19)$$

$$C_4(\rho_r, T_{pr}) = A_{10} (1 + A_{11} \rho_r^2) \left(\frac{\rho_r^2}{T_{pr}^3} \right) \exp(-A_{11} \rho_r^2) \quad (20)$$

where $A_1 = 0.3265$, $A_2 = -1.0700$, $A_3 = -0.5339$, $A_4 = 0.01569$, $A_5 = -0.05165$, $A_6 = 0.5475$, $A_7 = -0.7361$, $A_8 = 0.1844$, $A_9 = 0.1056$, $A_{10} = 0.6134$, and $A_{11} = 0.7210$.

The Dranchuk-Abou-Kassem EoS [9] must be solved iteratively since the Z-factor is on both sides of the equation. This task could be executed using the Newton-Raphson iteration method. Rearranging Equation 15 for this purpose yields:

$$F(Z) = Z - \{1 + C_1(T_{pr}) \cdot \rho_r + C_2(T_{pr}) \cdot \rho_r^2 - C_3(T_{pr}) \cdot \rho_r^5 - C_4 \rho_r T_{pr}\} \quad (21)$$

where

$$\left(\frac{\partial F(Z)}{\partial Z}\right)_{T_{pr}} = 1 + C_1(T_{pr}) \cdot \frac{\rho_r}{Z} + 2C_2(T_{pr}) \cdot \frac{\rho_r^2}{Z} - 5C_3(T_{pr}) \cdot \frac{\rho_r^5}{Z} + \left(\frac{2A_{10}\rho_r^2}{T_{pr}^2 Z}\right) \{1 + A_{11}\rho_r^2 - (A_{11}\rho_r^2)^2\} \exp(-A_{11}\rho_r^2) \quad (22)$$

The steps involved in evaluating Z are highlighted as follows:

1. Use Standing and Katz Z -factor Z_{SK} as initial guess Z^i
2. Evaluate the new improved estimate of Z as:

$$Z^{i+1} = Z^i - \frac{F(Z^i)}{F'(Z^i)} \quad (23)$$

where

$$F'(Z^i) = \left(\frac{\partial F(Z^i)}{\partial Z}\right)_{T_{pr}} \quad (24)$$

Repeat step 2 several times n until the error i.e. $abs(Z^n - Z^{n+1}) \approx 0$, such that $Z^n = Z^{n+1}$ at a convergence criterion of 10^{-4} .

The Newton-Raphson iteration method of determining the single-porosity Z -factors at different pressure levels without pore compaction was executed through a MAPPLE program.

2.2 Derivation of Free Gas Z-Factor for Single Porosity Gas Reservoirs with Rock Compaction

For single-porosity (no fracture) volumetric gas reservoirs (i.e. no water influx or water production), the MBE is as follows:

$$G_{mat}(B_g - B_{g_i}) + (G_{mat}B_{g_i})\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right)\Delta P + W_e = G_p B_g + W_p B_w \quad (25)$$

where G_{mat} is the OGIP in the matrix, B_{g_i} is initial formation volume factor, B_g is formation volume factor after pressure depletion ΔP , C_{matrix} is matrix compressibility, C_w is water compressibility, S_{w_i} is initial water saturation, C_{matrix} is matrix compressibility and S_{g_i} is initial gas saturation.

For volumetric gas reservoirs (no water influx or water production), the MBE is expressed as:

$$G_{mat}(B_g - B_{g_i}) + (G_{mat}B_{g_i})\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right)\Delta P = G_{p_{mat}} B_g \quad (26)$$

$$1 - \frac{B_{g_i}}{B_g} + \left(\frac{B_{g_i}}{B_g}\right)\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right)\Delta P = \frac{G_{p_{mat}}}{G_{mat}} \quad (27)$$

$$\frac{G_{p_{mat}}}{G_{mat}} = 1 - \frac{B_{g_i}}{B_g} \cdot \left\{1 - \left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right)\Delta P\right\} \quad (28)$$

But

$$\frac{B_{g_i}}{B_g} = \frac{Z_i P}{P_i Z} \quad (29)$$

Hence,

$$\left(\frac{G_p}{G}\right)_{Free} = 1 - \frac{P/Z}{P_i/Z_i} \cdot \left\{1 - \left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right)\Delta P\right\} \quad (30)$$

Equation 30 is the MBE developed for single-porosity gas reservoirs with rock compaction after pressure depletion, and the corresponding Z -factor is:

$$Z^* = Z \cdot \left\{1 - \left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right)\Delta P\right\}^{-1} \quad (31)$$

where Z is single-porosity Z -factor at pressure P without pore compaction and $\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right)$ is the single-porosity matrix pore volume compressibility. It is expressed here as $C_{p_{mat}}^*$.

2.3 Modification of Single-Porosity Free Gas Z-Factor to Dual-Porosity Free Gas Z-Factor

In this work, a simpler but accurate approach of developing dual porosity free gas Z -factor in MBE (that is different from the Aguilera [29] method) is considered. Incorporating the ratio of dual porosity to initial matrix porosity into single-porosity gas MBE yields dual-porosity MBE. The approach is as follows:

With reference to Equation 26, for dual porosity gas reservoirs,

$$G(B_g - B_{g_i}) + G\left(\frac{Dual\ Porosity}{Initial\ Matrix\ Porosity}\right)\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right) \cdot B_{g_i} \Delta P = G_p \cdot B_g \quad (32)$$

$$(B_g - B_{g_i}) + \left(\frac{Dual\ Porosity}{Initial\ Matrix\ Porosity}\right)\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right) \cdot B_{g_i} \Delta P = \frac{G_p}{G} \quad (33)$$

$$(B_g - B_{g_i}) + \left(\frac{\phi'_{mat}}{\phi'_{mat}} + \frac{\phi_{frac}}{\phi'_{mat}}\right)\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right) \cdot B_{g_i} \Delta P = \frac{G_p}{G} B_g \quad (34)$$

$$\frac{G_p}{G} = 1 - \frac{B_{g_i}}{B_g} + \left(\frac{B_{g_i}}{B_g}\right)\left(\frac{(1-\phi_{frac})\phi'_{mat}}{\phi'_{mat}} + \frac{\phi_{frac}}{\phi'_{mat}}\right)\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}}\right) \cdot \Delta P \quad (35)$$

$$\left(\frac{G_D}{G}\right)_{Free} = 1 - \frac{P/Z}{P_{i/Z_i}} \cdot \left\{ 1 - \left(1 - \phi_{frac} + \frac{\phi_{frac}}{\phi_{mat}} \right) \left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}} \right) \Delta P \right\} \quad (36)$$

and the developed free gas Z-factor for dual porosity is:

$$\left(\frac{G_D}{G}\right)_{Free} = 1 - \frac{P/Z}{P_{i/Z_i}} \cdot \left\{ 1 - \left(1 - \phi_{frac} + \frac{\phi_{frac}}{\phi_{mat}} \right) \left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}} \right) \Delta P \right\} \quad (37)$$

where Z is single-porosity Z-factor at pressure P without rock compaction.

2.4 Generalisation of the Developed Dual-Porosity Free Gas Z-Factor

Aguilera [29] dual-porosity free gas Z-factors is expressed as:

$$Z'' = Z \cdot \left\{ 1 - \left((1 - \omega_f) C_{p_{mat}}^* + \omega_f C_{p_{frac}}^* \right) \cdot \Delta P \right\}^{-1} \quad (38)$$

where Z is single-porosity Z-factor at pressure P without pore compaction, ω_f is fraction of OGIP (free gas) in the fracture system and $(1 - \omega_f)$ is fraction of OGIP (free gas) in the matrix, ΔP is pressure depletion, matrix pore volume compressibility $C_{p_{mat}}^*$ is expressed as $\left(\frac{C_w S_{w_i} + C_{matrix}}{S_{g_i}} \right)$ and fracture volume compressibility $C_{p_{frac}}^*$ is expressed as $\left(\frac{C_w S_{w_i} + C_{frac}}{S_{g_i}} \right)$.

However, from the concept of porosity, Aguilera [29] dual-porosity Z-factor is expressed in this work as:

$$Z'' = Z \cdot \left\{ 1 - \left(\left(\frac{\phi_{mat}}{\phi_{mat} + \phi_{frac}} \right) C_{p_{mat}}^* + \left(\frac{\phi_{frac}}{\phi_{mat} + \phi_{frac}} \right) C_{p_{frac}}^* \right) \cdot \Delta P \right\}^{-1} \quad (39)$$

where ϕ_{frac} is fracture porosity, $\phi_{mat}'' = (1 - \phi_{frac})\phi_{mat}'$ is matrix porosity after fracturing and ϕ_{mat}' is initial matrix porosity.

During hydraulic fractures stabilisation and porosity/permeability maintenance before pressure depletion, fracture volume compressibility ($C_{p_{frac}}^*$) is sustained at a lower level that corresponds to injection water compressibility ($C_{w_{Inj}}$). Hence, $C_{p_{frac}}^*$ is considered to be equal to $C_{w_{Inj}}$.

For the developed dual porosity free gas Z-factor to be generalised for hydraulically-fractured gas reservoirs, it is correlated with Aguilera [29] dual-porosity free gas Z-factor, and the qualities of fit were graphically and statistically assessed.

3. RESULTS AND DISCUSSION

Sampled gas compositions for shale formations A, B and C are shown in Tables 1, 2 and 3 respectively. The gas compositions in the Tables had been normalised to the reported compounds by Bullin and Krouskop [19], and Hill et al. [32].

Table 1. Gas composition for shale formation A

Well	Gas Composition (%)				
	C1	C2	C3	CO ₂	N ₂
1	79.4	16.1	4.0	0.1	0.4
2	82.1	14.0	3.5	0.1	0.3
3	83.8	12.0	3.0	0.9	0.3
4	95.5	3.0	1.0	0.3	0.2

Source: Bullin and Krouskop [19]; Hill et al. [32]

Table 2. Gas composition for shale formation B

Well	Gas Composition (%)				
	C1	C2	C3	CO ₂	N ₂
Average	95.0	0.1	0	4.8	0.1

Source: Bullin and Krouskop [19]; Hill et al. [32]

Table 3. Gas composition for shale formation C

Well	Gas Composition (%)				
	C1	C2	C3	CO ₂	N ₂
1	80.3	8.1	2.3	1.4	7.9
2	81.2	11.8	5.2	0.3	1.5
3	91.8	4.4	0.4	2.3	1.1
4	93.7	2.6	0.0	2.7	1.0

Source: Bullin and Krouskop [19]; Hill et al. [32]

In this work, Sutton correlation [14] ($0.57 < \gamma_g < 1.68$) is used in calculating pseudo-critical pressure P_{pc} and pseudo-critical temperature T_{pc} for shale formation A having average of 85.2% C1 see Table 1. However, Standing correlation [13] ($\gamma_g < 0.75$) is used in calculating P_{pc} and T_{pc} for shale formation B having average of 95.0 % C1 see Table 2 and shale formation C having average of 86.75% C1 see Table 3.

Reservoir data for shale formation A is shown in Table 4.

3.1 Variation of Natural Gas Compressibility Factor with Pressure for Shale Formation A

Reservoir temperature T of 175 °F (i.e. 635 °R) and a natural gas gravity γ_g of 0.65 are considered for shale formation A. Based on gas compositions (see Table 1), applying Sutton correlation [14] ($0.57 < \gamma_g < 1.68$) yields a pseudo-critical pressure P_{pc} of 670.13 psi (see Equation 5), pseudo-critical temperature T_{pc} of 365.11 °R (see Equation 6) and a pseudo-reduced temperature T_{pr} of 1.7392.

Standing and Katz Z-factors Z_{SK} are used as initial guesses in evaluating Dranchuk-Abou-Kassem Z-factors Z_{DAK} (see Equation 15) indicated as Z in Table 5. The pressure range considered for shale formation A is: $0 \leq P \leq 3,500$ psig. The initial gas formation volume factor of shale formation A:

$$B_{g_i} = \left(\frac{P_{sc}}{T_{sc}}\right) \frac{Z_i T}{P_i} \quad (40)$$

is evaluated as 4.6313×10^{-3} rcf/scf.

With $\phi_{frac} = 0$, single-porosity Z-factor with pore compaction is evaluated as:

$$Z^* = Z\{1 - (1.5429E - 6)\Delta P\}^{-1} \quad (41)$$

With $\phi_{frac} = 0.04$, Aguilera [27] dual-porosity Z-factor with pore compaction is evaluated as:

$$Z'' = Z\{1 - (2.1479E - 6)\Delta P\}^{-1} \quad (42)$$

and the modified dual-porosity Z-factor with pore compaction is evaluated as:

$$Z^{**} = Z\{1 - (2.0983E - 6)\Delta P\}^{-1} \quad (43)$$

The variations of the Z-factors with pressure depletion for shale formation A are shown in Table 5 and Fig. 1. It is observed that the single-porosity Z-factor, evaluated for shale formation A using the Dranchuk-Abou-Kassem eleven-constant equation of state (EoS) [9], decreases from 1.0000 (at $P = 0$), reaches its minimum value around 2,300 psig (i.e. around $P_{pr} = 3.4322$) and thereafter rises. The trend conforms to Standing and Katz chart within the pressure range considered but the higher decimal points yielded more accuracy than the Standing and Katz chart. Correlating the modified dual-porosity Z-factor with Aguilera [29] dual porosity Z-factor yields a R^2 value of 1.000.

Table 4. Reservoir data for shale formation A

Parameter	Symbol	Value	Unit
Matrix porosity before fracturing	ϕ'_{mat}	0.10	-
Fracture porosity	ϕ_{frac}	0.04	-
Initial gas saturation	S_{g_i}	0.70	-
Initial water saturation	S_{w_i}	0.30	-
Water compressibility	C_w	3.6×10^{-6}	psi ⁻¹
Rock matrix compressibility	C_{matrix}	4.0×10^{-12}	psi ⁻¹

Source: SPE Formation Evaluation [33]; US Department of Energy [34]; Soeder [35]

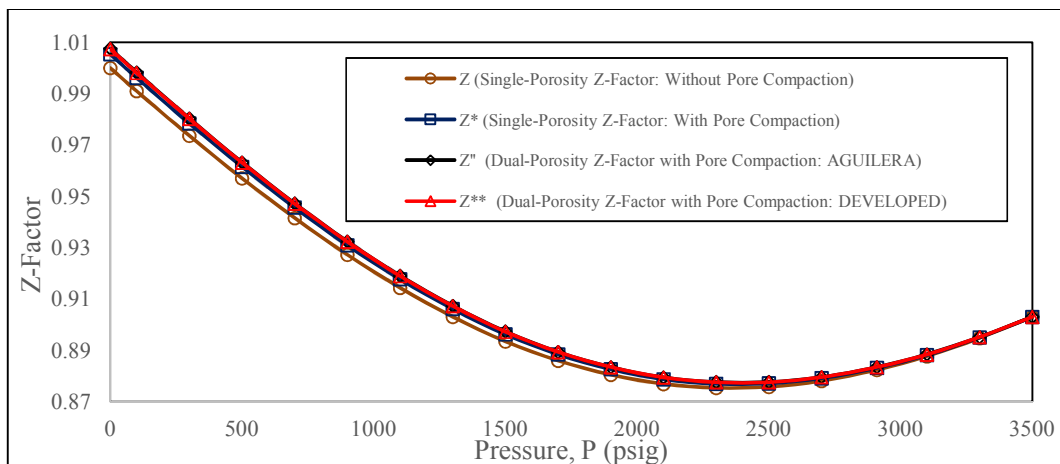


Fig. 1. Variation of natural gas Z , Z^* , Z'' and Z^{} with pressure for shale formation A based on ϕ_{frac} of 0.04**

Table 5. Variations of natural gas Z , Z^* , Z'' and Z^{} with pressure for shale formation A based on ϕ_{frac} of 0.04**

\square (psig)	P_{pr}	Z	ΔP (psig)	Z^*	Z'' (Aguilera)	\square^{**} (Developed)
3500	5.2229	0.9030	0	0.9030	0.9030	0.9030
3300	4.9244	0.8947	200	0.8950	0.8951	0.8951
3100	4.6260	0.8876	400	0.8881	0.8884	0.8883
2910	4.3424	0.8823	590	0.8831	0.8834	0.8834
2700	4.0291	0.8780	800	0.8791	0.8795	0.8795
2500	3.7306	0.8757	1000	0.8771	0.8776	0.8775
2300	3.4322	0.8753	1200	0.8769	0.8776	0.8775
2100	3.1337	0.8768	1400	0.8787	0.8794	0.8794
1900	2.8353	0.8804	1600	0.8826	0.8834	0.8834
1700	2.5368	0.8859	1800	0.8884	0.8893	0.8893
1500	2.2384	0.8934	2000	0.8962	0.8973	0.8972
1300	1.9399	0.9030	2200	0.9061	0.9073	0.9072
1100	1.6415	0.9143	2400	0.9177	0.9190	0.9189
900	1.3430	0.9272	2600	0.9309	0.9324	0.9323
700	1.0446	0.9415	2800	0.9456	0.9472	0.9471
500	0.7461	0.9570	3000	0.9615	0.9632	0.9631
300	0.4477	0.9736	3200	0.9784	0.9803	0.9802
100	0.1492	0.9910	3400	0.9962	0.9983	0.9981
0	0	1.0000	3500	1.0054	1.0076	1.0074

Reservoir data for shale formation B is shown in Table 6.

3.2 Variation of Natural Gas Compressibility Factor with Pressure for Shale Formation B

Reservoir temperature T of 300°F (i.e. 760°R) and a natural gas gravity γ_g of 0.65 are considered for shale formation B. Based on gas compositions (see Table 2), applying Standing [13] correlation ($\gamma_g < 0.75$) for dry gas yields a pseudo-critical pressure P_{pc} of 660.91 psi (see Equation 1), pseudo-critical temperature T_{pc} of 373.97 °R (see Equation 2) and a pseudo-reduced temperature T_{pr} of 2.0322.

Standing and Katz Z-factors Z_{SK} are used as initial guesses in evaluating Dranchuk-Abou-Kassem Z-factors Z_{DAK} (see Equation 15) indicated as Z in Table 7. The pressure range considered for shale formation B is: $0 \leq P \leq 12,000$ psig. The initial gas formation volume factor of shale formation B:

$$B_{gi} = \left(\frac{P_{sc}}{T_{sc}} \right) \frac{Z_i T}{P_i} \quad (44)$$

is evaluated as 2.1653×10^{-3} rcf/scf.

With $\phi_{frac} = 0$, single-porosity Z-factor with pore compaction is evaluated as:

$$Z^* = Z\{1 - (1.5429E - 6)\Delta P\}^{-1} \quad (45)$$

With $\phi_{frac} = 0.04$, Aguilera [27] dual-porosity Z-factor with pore compaction is evaluated as:

$$Z'' = Z\{1 - (2.2196E - 6)\Delta P\}^{-1} \quad (46)$$

and the modified dual-porosity Z-factor with pore compaction is evaluated as:

$$Z^{**} = Z\{1 - (2.2073E - 6)\Delta P\}^{-1} \quad (47)$$

The variations of the Z-factors with pressure depletion for shale formation B are shown in Table 7 and Fig. 2. Also, the single-porosity Z-factor calculated for shale formation B was found to decrease from 1.0000 (at $P = 0$), reaches its minimum value around 2,250 psig (i.e. around $P_{pr} = 3.4044$) and thereafter rises. As usual, the trend conforms to Standing and Katz chart within the pressure range considered but the higher decimal points showed more accuracy than the Standing and Katz chart. Correlating the modified dual-porosity Z-factor with Aguilera [29] dual porosity Z-factor yields a R^2 value of 1.000.

Reservoir data for shale formation C is shown in Table 8.

3.3 Variation of Barnett Shale Gas Compressibility Factor with Pressure

Reservoir temperature T of 180 °F (i.e. 640 °R) and a natural gas gravity γ_g of 0.65 are

considered for shale formation C. Based on gas compositions (see Table 3), applying Standing correlation [13] ($\gamma_g < 0.75$) for dry gas yields a pseudo-critical pressure P_{pc} of 660.91 psi (see Equation 1), pseudo-critical temperature T_{pc} of 373.97 °R (see Equation 2) and a pseudo-reduced temperature T_{pr} of 1.71140.

Standing and Katz Z-factors Z_{SK} are used as initial guesses in evaluating Dranchuk-Abou-Kassem Z-factors Z_{DAK} (see Equation 15) indicated as Z in Table 9.

The pressure range considered for shale formation C is: $0 \leq P \leq 3,900$ psig. The initial gas formation volume factor of shale formation C:

$$B_{g_i} = \left(\frac{P_{sc}}{T_{sc}}\right) \frac{Z_i T}{P_i} \quad (48)$$

is evaluated as 4.2614×10^{-3} rcf/scf.

With $\phi_{frac} = 0$, single-porosity Z-factor with pore compaction is evaluated as:

$$Z^* = Z\{1 - (1.5429E - 6)\Delta P\}^{-1} \quad (49)$$

With $\phi_{frac} = 0.02$, Aguilera [27] dual-porosity Z-factor with pore compaction is evaluated as:

$$Z'' = Z\{1 - (2.1847E - 6)\Delta P\}^{-1} \quad (50)$$

and the modified dual-porosity Z-factor with pore compaction is evaluated as:

$$Z^{**} = Z\{1 - (2.1978E - 6)\Delta P\}^{-1} \quad (51)$$

The variations of the Z-factors with pressure depletion for shale formation B are shown in Table 9 and Fig. 3. The single-porosity Z-factor evaluated for shale formation C was found to decrease from 1.0000 (at $P = 0$), reaches its minimum value around 2,300 psig (i.e. around $P_{pr} = 3.4800$) and thereafter rises. The trend conforms to Standing and Katz chart within the pressure range considered but the higher decimal points showed more accuracy than the Standing and Katz chart. Correlating the modified dual-porosity Z-factor with Aguilera [29] dual porosity Z-factor yields a R^2 value of 1.000.

Table 6. Reservoir data for shale formation B

Parameter	Symbol	Value	Unit
Matrix porosity before fracturing	ϕ_{mat}	0.085	-
Fracture porosity	ϕ_{frac}	0.04	-
Initial gas saturation	S_{g_i}	0.70	-
Initial water saturation	S_{w_i}	0.30	-
Water compressibility	C_w	3.6×10^{-6}	psi ⁻¹
Rock matrix compressibility	C_{matrix}	3.0×10^{-12}	psi ⁻¹

Source: US Department of Energy [34]; Male et al. [36]

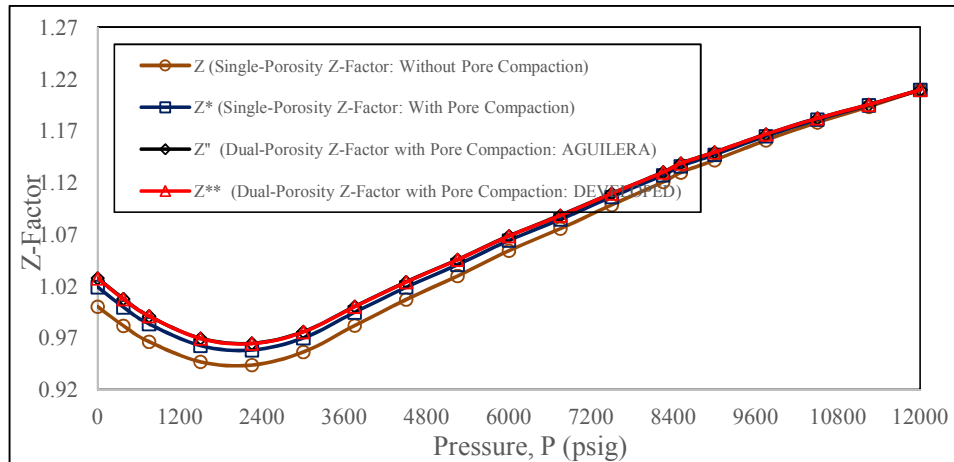


Fig. 2. Variations of natural gas Z, Z*, Z'' and Z with pressure for shale formation B based on ϕ_{frac} of 0.04**

Table 7. Variations of natural gas Z , Z^* , Z'' and Z^{} with pressure for shale formation B based on ϕ_{frac} of 0.04**

P (psig)	P_{pr}	Z	ΔP (psig)	Z^*	Z'' (Aguilera)	Z^{**} (Developed)
12000	18.1568	1.2094	0	1.2094	1.2094	1.2094
11250	17.0220	1.1931	750	1.1945	1.1951	1.1951
10500	15.8872	1.1780	1500	1.1807	1.1819	1.1819
9750	14.7524	1.1607	2250	1.1647	1.1665	1.1665
9000	13.6176	1.1415	3000	1.1468	1.1492	1.1491
8508	12.8732	1.1296	3492	1.1357	1.1384	1.1384
8250	12.4824	1.1205	3750	1.1270	1.1299	1.1298
7500	11.3480	1.0983	4500	1.1060	1.1094	1.1093
6750	10.2132	1.0754	5250	1.0842	1.0881	1.0880
6000	9.0784	1.0541	6000	1.0639	1.0683	1.0682
5250	7.9436	1.0297	6750	1.0405	1.0454	1.0453
4500	6.8088	1.0069	7500	1.0187	1.0239	1.0238
3750	5.6740	0.9818	8250	0.9945	1.0001	1.0000
3000	4.5392	0.9562	9000	0.9697	0.9757	0.9756
2250	3.4044	0.9436	9750	0.9580	0.9645	0.9644
1500	2.2696	0.9467	10500	0.9623	0.9693	0.9692
750	1.1348	0.9661	11250	0.9832	0.9908	0.9909
375	0.5674	0.9815	11625	0.9994	1.0075	1.0073
0	0	1.0000	12000	1.0189	1.0274	1.0273

Table 8. Reservoir data for shale formation C

Parameter	Symbol	Value	Unit
Matrix porosity before fracturing	ϕ'_{mat}	0.045	-
Fracture porosity	ϕ_{frac}	0.02	-
Initial gas saturation	S_{g_i}	0.70	-
Initial water saturation	S_{w_i}	0.30	-
Water compressibility	C_w	3.6×10^{-6}	psi ⁻¹
Rock matrix compressibility	C_{matrix}	3.0×10^{-12}	psi ⁻¹

Source: US Department of Energy [34]; Bowker [37]

Table 9. Variations of natural gas Z , Z^* , Z'' and Z^{} with pressure for shale formation C based on ϕ_{frac} of 0.02**

P (psig)	P_{pr}	Z	ΔP (psig)	Z^*	Z'' (Aguilera)	Z^{**} (Developed)
3900	5.9009	0.9186	0	0.9186	0.9186	0.9186
3600	5.4470	0.9020	300	0.9024	0.9026	0.9026
3300	4.9931	0.8878	600	0.8886	0.8890	0.8890
3050	4.6148	0.8784	850	0.8795	0.8800	0.8800
2800	4.2366	0.8714	1100	0.8729	0.8735	0.8735
2550	3.8583	0.8671	1350	0.8689	0.8697	0.8697
2300	3.4800	0.8660	1600	0.8681	0.8690	0.8691
2088	3.1593	0.8676	1812	0.8700	0.8710	0.8711
1800	2.7235	0.8738	2100	0.8766	0.8778	0.8779
1550	2.3452	0.8830	2350	0.8862	0.8876	0.8876
1300	1.9670	0.8955	2600	0.8991	0.9006	0.9006
1050	1.5887	0.9110	2850	0.9150	0.9167	0.9167
800	1.2104	0.9292	3100	0.9337	0.9355	0.9356
550	0.8322	0.9495	3350	0.9544	0.9565	0.9565
300	0.4539	0.9717	3600	0.9771	0.9794	0.9794
150	0.2270	0.9856	3750	0.9913	0.9937	0.9938
0	0	1.0000	3900	1.0060	1.0086	1.0086

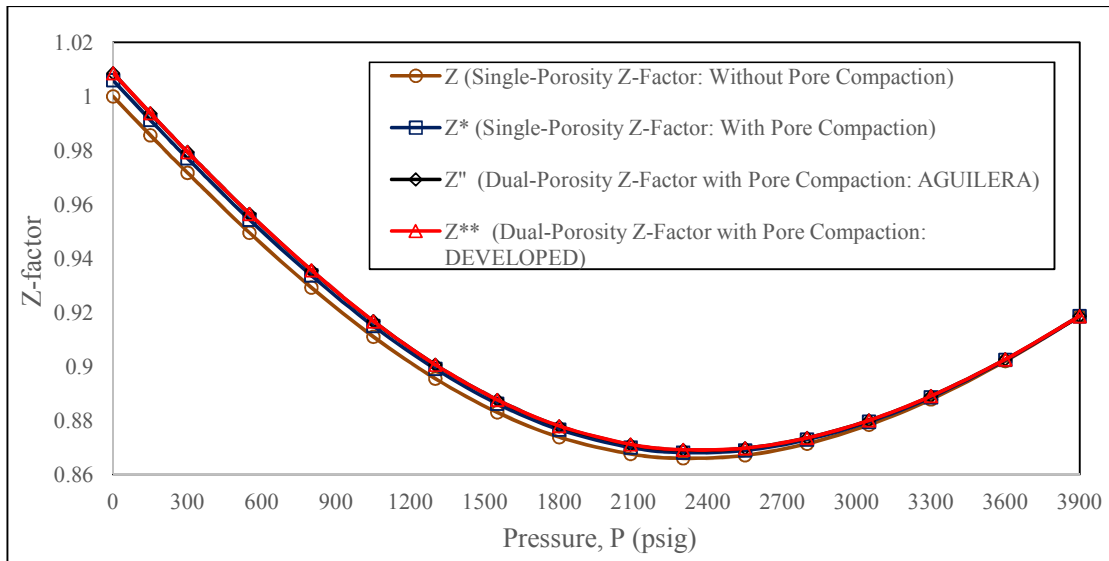


Fig. 3. Variations of natural gas Z , Z^* , Z'' and Z^{} with pressure for shale formation C based on ϕ_{frac} of 0.02**

4. CONCLUSIONS

The, free gas Z-factor in the Material Balance Equation (MBE) of single-porosity gas reservoirs with insignificant rock (matrix) compaction (after pressure depletion) does not reflect cases in low-permeability gas reservoirs having remarkable rock compaction. Therefore, in this work, a free gas Z-factor for single-porosity reservoirs with remarkable pore compaction (such as low-permeability gas reservoirs) was derived and it is expressed as: $Z^* = Z \cdot \left\{ 1 - \left(\frac{C_w S_{wi} + C_{matrix}}{S_{g_i}} \right) \Delta P \right\}^{-1}$ where Z , C_w , S_{wi} , C_{matrix} and S_{g_i} are single-porosity Z-factor at P without rock compaction, water compressibility, initial water saturation, matrix compressibility and initial gas saturation, respectively.

However, Z^* was further modified to a simpler but accurate dual-porosity free gas Z-factor which incorporates ratio of dual porosity to initial matrix porosity, and it is expressed as: $Z^{**} = Z \cdot \left\{ 1 - \left(1 - \phi_{frac} + \frac{\phi_{frac}}{\phi_{mat}} \right) \left(\frac{C_w S_{wi} + C_{matrix}}{S_{g_i}} \right) \cdot \Delta P \right\}^{-1}$ where ϕ_{mat} and ϕ_{frac} are initial matrix porosity and fracture porosity respectively.

It is observed that the single-porosity Z-factor, evaluated for shale formations considered using the Dranchuk-Abou-Kassem eleven-constant equation of state (EoS) decreases from 1.0000

(at $P = 0$), reaches a minimum value and thereafter rises. The trend conforms to Standing and Katz chart within the pressure (and pseudo-pressure) range considered but the higher decimal points yielded more accuracy than the Standing and Katz chart.

The developed dual porosity free gas Z-factor was graphically and statistically correlated with Aguilera [29] dual porosity free gas Z-factor. For all the hydraulically fractured shale gas formations considered, the correlations yield R^2 values of 1.000.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

Derivation of Aguilera (2008) Dual-Porosity Free Gas Z-Factor

For fractured gas reservoirs without adsorption, the derivation of Aguilera (2008) gas MBE is as follows:

For volumetric gas reservoirs (no water influx or water production), the MBE is expressed as:

$$G_{mat} (B_g - B_{g_i}) + (G_{mat} B_{g_i}) \left(\frac{C_w S_{wi} + C_{pm}}{s_{g_i}} \right) \Delta P = G_{p_{mat}} B_g \quad (A.1)$$

For the matrix blocks in the dual-porosity system,

$$G_{mat} (B_g - B_{g_i}) + (G_{mat} B_{g_i}) \left(\frac{C_w S_{wm} + C_{pm}}{(1 - S_{wm})} \right) \Delta P = G_{p_{mat}} B_g \quad (A.2)$$

where G_{mat} , $G_{p_{mat}}$ and $\left(\frac{C_w S_{wm} + C_{pm}}{(1 - S_{wm})} \right)$ are OGIP within the matrix, cumulative gas production from the matrix and matrix pore volume compressibility respectively.

However, for the fractures,

$$G_{frac} (B_g - B_{g_i}) + (G_{frac} B_{g_i}) \left(\frac{C_w S_{wf} + C_f}{(1 - S_{wf})} \right) \Delta P = G_{p_{frac}} B_g \quad (A.3)$$

where G_{frac} , $G_{p_{frac}}$, $\left(\frac{C_w S_{wf} + C_f}{(1 - S_{wf})} \right)$ are OGIP within the fractures, cumulative gas production from the fractures and fracture volume compressibility respectively.

Expressing $\left(\frac{C_w S_{wm} + C_{pm}}{(1 - S_{wm})} \right)$ and $\left(\frac{C_w S_{wf} + C_f}{(1 - S_{wf})} \right)$ as C' and C'' respectively, and adding **Equations A.2** and **A.3** to obtain dual-porosity gas MBE (i.e. after fracturing) gives:

$$(G_{mat} + G_{frac}) (B_g - B_{g_i}) + (G_{mat} C' + G_{frac} C'') B_{g_i} \Delta P = (G_{p_{mat}} + G_{p_{frac}}) B_g \quad (A.4)$$

However, G (OGIP) is constant, i.e. $G = G_{mat} + G_{frac}$. Also, $G_{p_{mat}} + G_{p_{frac}} = G_p$. Hence,

$$G (B_g - B_{g_i}) + (G_{mat} C' + G_{frac} C'') B_{g_i} \Delta P = G_p B_g \quad (A.5)$$

$$(B_g - B_{g_i}) + \left(\frac{G_{mat}}{G} C' + \frac{G_{frac}}{G} C'' \right) B_{g_i} \Delta P = \frac{G_p}{G} B_g \quad (A.6)$$

$$(B_g - B_{g_i}) + ((1 - \omega_f) C' + \omega_f C'') B_{g_i} \Delta P = \frac{G_p}{G} B_g \quad (A.7)$$

where ω_f is fraction of OGIP (free gas) in the fracture system and $(1 - \omega_f)$ is fraction of OGIP (free gas) in the matrix.

$$\left(\frac{G_p}{G} \right)_{Free} = 1 - \frac{B_{g_i}}{B_g} + \left((1 - \omega_f) C' + \omega_f C'' \right) \frac{B_{g_i}}{B_g} \Delta P \quad (A.8)$$

Hence,

$$\left(\frac{G_p}{G} \right)_{Free} = 1 - \frac{P/z}{P_i/z_i} \cdot (1 - (1 - \omega_f) C' + \omega_f C'') \Delta P \quad (A.9)$$

where the modified Z-factor (Z'') is expressed as:

$$Z'' = Z \cdot \left\{ 1 - \left((1 - \omega_f)C' + \omega_f C'' \right) \cdot \Delta P \right\}^{-1} \quad (\text{A.10})$$

where Z is single-porosity Z-factor at pressure P without rock compaction. Equation A.9 is the Aguilera (2008) gas MBE for fractured gas reservoirs without adsorption and Equation A.10 is the Aguilera (2008) dual-porosity Z-factor.

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